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THE AIRFLOW OVER MOUNTAINS

A REPORT ON THE STATE OF CURRENT KNOWLEDGE

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SUMMARY

A brief survey is first given of the observational evidence regarding special airflow effects in the neighbourhood of mountains, as provided by the visual evidence of clouds, the experiences of glider pilots and effects noted by the pilots of powered aircraft. This serves to indicate the phenomena requiring explanation. The review of theoretical studies is confined mainly to the work of Queney and Scorer. Both these authors apply the perturbation method to laminar, isentropic and inviscid flow over an ideal ridge. Queney deals with a uniform airstream (constant velocity and constant stability) and derives streamlines showing wave motions, both vertical and horizontal. Queney's waves are dependent partly on the stability and partly on geostrophic forces, and the order of the wavelength is several hundred km. In his treatment, Scorer deals with smaller scale phenomena, so enabling geostrophic forces to be neglected, and provides for variations of stability and wind shear in the vertical. He finds that realistic wave effects, of the type observed in nature, are predicted by the theory provided the variation of stability and wind shear with height is in accord with certain criteria. The criteria have some practical value for forecasting purposes. Past

/experimental

experimental work with models in wind tunnels is briefly considered in the light of what can be deduced from theoretical treatments regarding the conditions for dynamical similarity. More recent work by Long, using a tank containing several layers of immiscible liquid to simulate a density gradient is referred to briefly; this technique appears to have promise. A comprehensive field study by Förchtgött, who used teams of gliders, is summarised. From his observations, Förchtgott has built up a theory of the flow patterns which occur in the lee of mountains. The main value of Förchtgott's work lies in the practical support it provides for some of the results deduced by perturbation theory and in throwing light on the nature of non-laminar flow patterns which cannot be dealt with by existing theoretical treatments. In a concluding section, aviation applications of existing knowledge on this subject are discussed. These concern forecasting and the determination of safety heights over mountains.

INTRODUCTION

It has long been known that the airflow over mountainous terrain is commonly more disturbed than over level country. In the past, information on the subject in meteorological textbooks has usually been scanty and generally confined to the statement that disturbances to the airstream above a given mountain can be expected to extend to about three times its height. This gives no clue as to the nature of the disturbances or the meteorological factors which have a bearing on them. From its very incompleteness, therefore, the information is quite unsatisfactory especially now, in view of the increasing demand for more detailed advice on the subject and the better understanding of such problems which has developed recently.

It might be supposed that the effect of a mountain range on an airstream would be merely to bring about a change in the extent, scale and degree of turbulence experienced by an aircraft flying above. Turbulence, however, is essentially a disorganised type of flow whilst often the airstream over a mountain is deformed in a fully organised manner which is highly significant but not necessarily turbulent. In this report we shall be much concerned with such organised deformations and the circumstances in which they occur.

During the last few years, there has grown a considerable body of experimental data on the subject from various sources and notable theoretical researches into the problem have been carried out, so much so that over at least an important part of the subject a logical system of ideas has developed and can now be presented. As glider pilots rely on vertical currents to support their sail-planes, it is natural that they should have learnt a great deal of the special airflow effects which occur in the neighbourhood of mountains; accordingly, much of the practical evidence has come from glider pilots. Förchtgott has exploited the potentialities of gliders in a valuable and illuminating programme of field research. More recently the experiences of the pilots of powered aircraft have been receiving attention and have been reported by Austin, Turner and others. As regards other specific experimental work, mention must be made of the investigation by Field and Warden (1929-30) into the air currents in the Bay of Gibraltar, the work of Abe in Japan who studied mountain cloud formations and supplemented this by wind tunnel experiments, and Küttner's studies of the airflow over the Alps. Currently, the powerful mountain wave phenomena at Bishop, California, are the subject of an elaborate investigation as part of the U.S. Research project on this subject.

The challenge to the mathematician offered by the problem of mountain airflow has been taken up, and major contributions in the form of theoretical treatments have been made by Lyra, Queney, Scorer and others. The hydrodynamical theory solves the problem of the flow pattern successfully if the flow is laminar and steady, but when this is not the case we must draw what

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conclusions we can from the observational data. The theory indicates the great limitations of wind tunnel experiments and makes it clear that the problem of ensuring dynamical similarity in the general case poses almost insuperable laboratory difficulties; any past results obtained in this way, therefore, must be interpreted with great caution.

It is the purpose of this report to summarise existing knowledge on the subject of mountain airflow. The literature is very extensive and we shall confine ourselves to those aspects which appear to have a bearing, real or potential, on the aviation problems arising from the subject.

PART I - SUMMARY AND EXAMPLES OF OBSERVATIONAL EVIDENCE

The observational evidence of special airflow effects in the neighbourhood of mountains, notably the phenomenon of stationary lee waves, falls roughly into three categories, viz. the visual evidence provided by clouds, the experiences of glider pilots, and effects observed from powered aircraft. It will be convenient, before discussing theoretical and other studies, to summarise this evidence and give a few examples, in order to make clear the nature of the phenomena requiring explanation.

(a) The Evidence of Mountain Clouds

Lenticular clouds are frequently seen over hilly or mountainous terrain in all parts of the world. As observed from the ground they have smooth lens-shaped outlines and remain stationary relative to the ground or nearly so. The wind blows through them and this can be verified when the movement of an identifiable fragment of the cloud can be followed visually. This has been confirmed by time lapse photography of mountain clouds, for example by Abe (1). The inference that such clouds form where the airstream executes a vertical undulation initiated by the terrain below is almost inescapable. On many occasions, especially to the lee of long ridges, a succession of lenticular clouds, parallel to each other and to the ridge, may be seen; the suggestion that they constitute visible signs of systems of atmospheric lee waves is very strong. Over irregular hilly terrain, the lenticulars too are irregularly distributed; such skies are frequent in Scotland and many examples are to be found in the Clarke collection of cloud photographs.

A similar phenomenon which can sometimes be observed occurs when an airstream containing a layer of stratocumulus crosses a mountainous area. Organised stationary clearances of the cloud in the form of holes or parallel zones to the lee of hills or ridges can be seen. The inference is that the clearances are located where the airstream executes downward undulations associated with the topography.

A number of field studies have been carried out aimed at elucidating the mechanism of formation of particular cloud formations which are characteristic of given hills or mountains. Amongst these are Manley's exhaustive study (2) of the Crossfell Helm wind, and its associated clouds, and Küttner's study (3) of the so-called Moazagotl clouds which form over the Reisengebirge in the Alps. Although on a different scale, both these phenomena have much in common and in particular at both locations rotor clouds are commonly observed at levels similar to those of the hilltops, whilst lenticular clouds may also be seen high above the rotors.

/Recently

Recently Ludlam (4) has discussed the formation of orographic cirrus over the hills of U.K. and has put forward evidence which strongly supports his suggestion that a great deal of cirrus forms in atmospheric waves initiated by topographical features. He is satisfied from his observations that hills in U.K. can produce waves having sufficient amplitude at 20,000 feet or more to produce cirrus cloud. This comes as no surprise to those who have accepted that 'mother of pearl' clouds seen high above the Norwegian mountains at 70,000-80,000 feet and described by Störmer (5) also owe their origin to topography. 'Mother of pearl' clouds have been observed in other parts of the world besides Norway, even it seems over Scotland on rare occasions.

(b) The Experiences of Glider Pilots

In the early days of gliding, the sources of vertical motion made use of were first the upslope motion to be found on the windward side of hills and ridges and later thermal up-currents. During the 1930's, the possibilities of soaring in atmospheric waves began to be exploited, especially on the continent. Numerous ascents were made on the North side of the Alps in Föhn wind conditions and by 1939 several glider ascents to beyond 30,000 feet had been made in the air rising on the upwind side of wave clouds. The significant aspect was that many of these ascents were made well to the lee of the highest ground and could not, therefore, have been simple cases of flights in the airstream ascending the windward slopes. The first important ascent by a glider in a standing wave in U.K. was made in 1939 by McLean who reached 11,000 feet on the Helm phenomenon. On this first occasion, McLean had difficulty in getting down and was only able to do so by locating the downdraught of the wave. The occurrence was described graphically by Horsley (6).

Since the war, wave soaring has become commonplace and the various Gliding journals literally abound with articles by glider pilots describing experiences in standing waves. Some of these accounts are merely descriptive but many others are based on careful and detailed observation. The sinking speed of a glider for a given forward speed is a quantity which is known fairly exactly, especially if the machine has been calibrated, and therefore gliders are able to explore waves and determine the vertical component of the wind. It is by such means that a fairly complete picture of the characteristics of standing lee waves has emerged.

From large numbers of reports of this sort has grown the conception of standing waves as vertical undulations in the airstream to the lee of hills and ridges. It is found that over the lee ground, waves, if any, increase in amplitude upwards to some level above which the waves decrease and eventually die out. They die away downstream but a succession of several observable waves has often been noted in U.K. The order of magnitude of the wavelength is usually in the range 1 to 10 miles, but wavelengths up to 20 miles or so are sometimes observed.

We can scarcely leave this reference to gliders without mentioning the spectacular 'Bishop' wave which is characteristic of the airstream over the Sierra Nevada range in California during winter time. Many ascents to well above 40,000 feet have been made by gliders in this wave system and there seems no reason why a pressurised glider, which is planned, should not reach far greater heights. If the wave develops at all, vertical components of 2,000 ft./min. are common, whilst 4,000 ft./min. has been recorded and it is believed that components up to 5,000 ft./min. may occur on occasions. The field

/work

work for the U.S. Mountain Wave Project was carried out at this location, but reports published so far have been confined to descriptions of the organisation of the project and of the techniques used for tracking gliders, etc. A reasoned discussion of the results obtained has yet to appear.

(c) Effects Observed from Powered Aircraft

During the last 3-4 years there has been a steadily increasing number of reports of mountain airflow effects from the pilots of powered aircraft. This may be because aircraft are now flying over mountain ranges which lie across air routes, with increasing regularity and frequency. The reports confirm that areas of lift and sink are commonly to be found over and to the lee of mountains. To date most of these reports have been made by pilots who have for some reason a special interest in the subject as, for example, because they are also glider pilots, or because a specific experience was sufficiently notable to arouse a special interest. Mild standing lee waves may easily pass unnoticed in a powered aircraft because they are not necessarily accompanied by any turbulence effects and in fact many reports mention the remarkably smooth flying conditions in waves. In addition, a pilot ignorant of wave effects might well ascribe gentle height fluctuations, having a period of a minute or two, to oscillations of the aircraft due to instability of trim or to the movement of passengers about the cabin.

Austin and Turner have reported in references (7), (8), (9) cases of standing waves encountered by pilots. Furthermore, the Aviation journals contain several contributions by pilots describing their experiences. One pilot, Capt. Neilan of Skyways, whose work amongst other things involves the flight testing of aircraft over Scotland, has reported details of several experiences in standing waves. On many occasions Capt. Neilan has encountered alternate areas of up and down currents exceeding 1,000 ft./min.; such currents force themselves upon his attention because they are a hindrance to flight testing.

Capt. Mason of B.E.A. in an article in Shell Aviation News (10) gave a detailed description of an incident over the mountains of Northern Spain on 18th December 1952, when he was flying a Viking aircraft from Madrid to London. He approached the mountains north of Madrid at 11,000 feet and the aircraft subsequently fell to 8,800 feet and then rose to 14,000 feet. This was repeated three times in what were undoubtedly powerful standing waves. The height changes took place in spite of his using maximum power in the areas of sink and almost closing the throttles in the areas of rising air. It can be inferred from this that the vertical components exceeded 2,500 ft./min.

Some fifty or so reports of mountain airflow effects have now been collected at Northolt from British European Airways pilots and a record of these has been assembled by Turner in an unpublished report. Three cases from this record are now quoted by way of example.

(i) 18th December 1952
2000 G.M.T.

Mean flying height 8,500 feet. Location: midway between Nice and Montelimar. After slight vertical motion two powerful waves were encountered giving lift of 1,000 ft. per minute with aircraft in descending attitude and sink of 1,000 ft. per minute with full climbing power on. Wave crests about 8 minutes apart. Wind 340 deg. 30 kt. Sky cloudless.

/(ii)

(ii) 21st March 1952
1210-1310 G.M.T.

Mean flying height 10,500 feet. Location: Maritime Alps into Rhone Valley. Successive up and down draughts of 1,500 feet per minute resulting in 2,000 feet displacement of aircraft. There were 3/8 Cu below aircraft, otherwise no cloud. Wind NW. 40 kt.

(iii) 11th March 1953
1030 G.M.T.

Mean flying height 5,500 feet. Location: Barrow-in-Furness to Solway. Smooth waves encountered giving up and down currents of 700 feet per minute. Pilot made no attempt to counteract vertical motion, and therefore there were only small fluctuations of airspeed. About six separate waves were distinguished but beyond Solway the waves became negligible. Cloud: 6/8 Sc below aircraft and 7/8 Ac above. Wind 350 deg. 35-40 kt.

One feature worth noting is that in many cases the pilot records a marked drop in airspeed on entering an area of sink and an increase in airspeed in the areas of lift. This, of course, would be expected if the pilot uses his controls in an attempt to maintain constant height, but it is a little unexpected to find similar (although smaller) variations of airspeed when no such correcting action is taken. This aspect has been discussed from the aerodynamical standpoint by Yates (11).

Turner examined the distribution of lapse-rate and wind in the vertical in the cases reported at Northolt, with a view to ascertaining whether the airstreams possessed any special characteristics common to all cases. His conclusions were used to carry out a forecasting test which was reported in M.R.P. 815. Briefly, he found that when pilots noted special airflow effects over hilly terrain, there was always a markedly stable layer (commonly at least isothermal) at middle levels, with usually a layer of steep lapse rate below. The type of effect depended upon the wind shear. When standing waves were found, the vertical wind shear was in the direction of the general wind (i.e. the wind increased with height). In the cases of marked turbulence the vertical wind shear was opposed to the general wind, i.e. there was reversed wind shear. In some intermediate cases both turbulence and standing waves, sometimes at different levels, occurred. It cannot be held, of course, that this kind of empirical approach yields conclusive evidence of necessary conditions for given effects, but it is interesting to note that although based on few observations Turner's conclusions are broadly consistent with some of the results obtained by Scorer and Förchtgott and described later in this report. With regard to the lower layer of steep lapse rate, found by Turner, one might expect this to be a normal attribute of a fast-moving airstream over rough hilly terrain. However, he found the layer to be at least 5,000 feet thick and it may, therefore, have real significance; in addition, there are other grounds against thinking this factor to be merely incidental and some notes by Corby on this point were added to M.R.P. 815. Turner's views developed from his own personal enquiries of pilots in the course of aviation forecasting, and his results showed that useful information may be obtained in this way. In the hope of securing further information more suitable for investigation purposes, arrangements have now been made at Northolt for a regularised scheme of detailed reporting by Northolt-based B.E.A. pilots.

/PART II

PART II - THEORETICAL STUDIES

(Note: For convenience in this section a common notation is used throughout which is not necessarily that of the particular author being quoted. After Queney we shall use $\rightleftharpoons$ to denote "is equal to the real part of".)

(a) General

The problem of the flow of an incompressible, homogeneous and inviscid current over an obstacle at the bottom was first solved by Kelvin in 1886. Lamb extended this work in 1916 to cover the case of a stratified, but still incompressible, fluid. In their solutions a system of stationary gravity waves appears downstream of the obstacle, provided the speed of the flow is less than a critical value depending on the depth of the fluid. These waves are analagous to those which constitute the wash of a ship.

The corresponding atmospheric problem is a great deal more complex. Firstly, apart from shallow adiabatic layers, e.g. near the ground on hot days, the atmosphere has continuous stable stratification. Secondly, the atmosphere, being compressible, has no definite upper boundary; as we shall see later the formulation of a satisfactory upper boundary condition in the atmospheric problem is difficult and has given rise to some controversy. Finally, for phenomena on the scale of, say, the airflow over the Alps, the earth's rotation must be taken into account, whilst on an even larger scale the variation of Coriolis parameter with latitude must be introduced.

So far, all theoretical treatments have made use of the perturbation method since no other mathematical method yet exists which can be applied successfully to this problem. In this method, the motion is considered as a disturbance superimposed on a given basic flow which is assumed to be laminar, frictionless and isentropic in the first instance. The hydro-dynamical equations of motion are set up and are used with the equation of continuity and the equation expressing the isentropic nature of the flow. It is assumed that the disturbance quantities are small compared with the values in the basic motion and this justifies neglect of the squares and products of the disturbance quantities and enables the equations to be linearised. Elimination of all but one of the unknowns yields a partial second order differential equation which is used to determine the nature of the perturbation.

(b) The work of Queney

We shall first consider the work of Queney who commenced studying the subject in 1936. After several contributions he published in 1947 a completely general theory of adiabatic perturbations in a stratified rotating atmosphere (12). The problem of the flow of an airstream over a mountain is a particular application of his generalized theory. Queney found that the degree of stability and the vertical component of the earth's rotation made the important contributions to the terms in the wave equation and he was able to justify neglect of several other terms. The detail of Queney's mathematics is elaborate and lengthy. For brevity in this report and for convenience in comparing subsequently Queney's results with Scorer's we may regard Queney's treatment as equivalent to the derivation of the wave equation represented by:

$$\left(1-\frac{f^2}{K^2U^2}\right)\frac{\partial^2\psi}{\partial z^2}-\left(\frac{g}{c^2}+\beta\right)\frac{\partial\psi}{\partial z}+\left(\frac{g\beta}{U^2}-K^2\right)\psi=0 \quad \ldots(1)$$

/where

where U = horizontal velocity of the undisturbed current (assumed constant and uniform in space)

ψ = stream function of the disturbance

f = 2 x vertical component of the earth's angular velocity

g = acceleration due to gravity

β = coefficient of stability defined as $\frac{1}{\theta}\frac{\partial \theta}{\partial z}$ where θ is potential temperature

K = wave number of the disturbance in the x direction (the direction of U) i.e. a harmonic variation of the disturbance is assumed in the x direction of wave-length $2\pi/K$

c = velocity of sound

z = height measured vertically upwards.

This equation expresses the variation of the disturbance with height assuming that the variation in the horizontal is harmonic. The nature of the source of the perturbation is as yet unspecified. Queney introduces a further simplification by taking the stability as constant with height and is then able to treat the coefficients of Eq.(1) as sensibly constant (for a given value of K) provided the magnitude of the perturbation is not of the order of thousands of km.

$\sqrt{\frac{g\beta}{U}}$ is denoted by l_s and $\frac{f}{U}$ by l_f

and we put $$\mu = +K\sqrt{\frac{l_s^2 - K^2}{l_f^2 - K^2}}$$

The solution of Eq. (1) may then be written

$$\psi = A e^{\lambda_1 z} + B e^{\lambda_2 z}$$

where $$\lambda_1, \lambda_2 = \frac{1}{2}\left\{\frac{K^2(g/c^2+\beta)}{K^2 - l_f^2} \pm \sqrt{\left(\frac{K^2(g/c^2+\beta)}{K^2 - l_f^2}\right)^2 + 4\mu^2}\right\}$$

$$\doteq \frac{1}{2}\left(g/c^2 + \beta\right) \pm \mu$$

if K is not in the vicinity of

l_f or l_s.

/Also

Also if ρ = density, $\left(g/c^2 + \beta\right) = -\frac{1}{\rho}\frac{\partial \rho}{\partial z}$

$= 2S$, say

(Queney calls S the heterogeneity)

Thus the solution may be expressed as

$$\psi = e^{Sz}\left(Ae^{\mu z} + Be^{-\mu z}\right) \text{ - - - - - - (2)}$$

Queney shows that with typical atmospheric values, e^{Sz} is closely equal to $\sqrt{\frac{\rho_0}{\rho}}$.

The vertical displacement of the streamlines from their undisturbed level will then be given by

$$\zeta = \frac{\psi}{U} = \frac{1}{U}\sqrt{\rho_0/\rho}\left(Ae^{\mu z} + Be^{-\mu z}\right) \text{ - - (3)}$$

We have the two cases:

(i) If μ is real, i.e. if $\kappa > \ell_s$ or $\kappa < \ell_f$ we must have A = 0, otherwise the disturbance would increase exponentially upwards indefinitely (clearly an unacceptable solution because such a result is impractical).

(ii) If μ is imaginary, i.e. if $\ell_s > \kappa > \ell_f$ the solution becomes

$$\zeta = \frac{1}{U}\sqrt{\rho_0/\rho}\left(Ae^{i\nu z} + Be^{-i\nu z}\right) \text{ where } \mu = i\nu.$$

This represents sinusoidal variation in the vertical. Relative to the airstream there is a downward progressing wave component $e^{i\nu z}$ and an upward component $e^{+i\nu z}$. Both are indeterminate for $Z = \infty$ and so there is some difficulty about the upper boundary condition. Queney overcomes the difficulty by temporarily introducing small viscosity (an artifice used by Rayleigh and others in problems of fluid motion). He does this by applying a factor $e^{-\epsilon z}$ to the term in $e^{i\nu z}$ and $e^{+\epsilon z}$ to the term in $e^{-i\nu z}$ where ϵ is small and positive. He is then, of course, able to reject the upward progressing wave and takes as his solution for imaginary μ :

$$\zeta = \frac{A}{U}\sqrt{\rho_0/\rho}\, e^{i\nu z}.$$

/The

The second boundary condition at the ground causes no difficulty. If the ground profile is the sinusoidal corrugation represented by the real part of $e^{i\kappa x}$ then we must have $\zeta = e^{i\kappa x}$ for $z = 0$. Thus the appropriate solutions for such a ground profile are:

For $\kappa > \ell_s$ or $\kappa < \ell_f$,

$$\zeta = \sqrt{\ell_0/\ell}\, e^{i\kappa x - \mu z}$$

$$= \sqrt{\ell_0/\ell}\, e^{-\mu z} \cos \kappa x \quad \text{---------- (4)}$$

For $\ell_s > \kappa > \ell_f$,

$$\zeta = \sqrt{\ell_0/\ell}\, e^{i(\kappa x + \nu z)}$$

$$= \sqrt{\ell_0/\ell}\, \cos(\kappa x + \nu z) \quad \text{---------- (5)}$$

Similarly, expressions could be deduced for the horizontal disturbance and the variation of pressure or density. The concept of a sinusoidal ground profile extending to infinity does not in itself provide a solution of direct practical value but it is derived in anticipation of subsequent generalisation by the Fourier integral method. Before doing so it is worth examining the elementary solutions given by (4) and (5) and in Fig. 1 are illustrated the streamlines corresponding to steady flow over an infinite sinusoidal ground profile for the three cases $\kappa > \ell_s$, $\ell_s > \kappa > \ell_f$ and $\kappa < \ell_f$.

For $\kappa > \ell_s$, (Fig. 1a), the order of the wavelength of the ground profile, with typical atmospheric values, is up to about 2π km. We see that on this scale, the solution gives wavy streamlines whose amplitude decreases exponentially upwards. The critical wavelength up to which this solution applies is determined by the value of ℓ_s i.e. by the stability and speed of the airstream. In the atmosphere $\ell_s \gg \ell_f$ and on this scale ℓ_f may be ignored, which is equivalent to saying that geostrophic forces have no opportunity to affect the result. There is thus negligible horizontal deformation in this case. The restoring force which makes the wave motion in the vertical possible arises solely from the stratification of stability, represented by ℓ_s.

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For $l_s > \kappa > l_f$, (Fig. 1b), the order of the wavelength of the ground profile is in the approximate range 2π to 200π km. On this scale geostrophic forces as well as stability play a part and there is both vertical and horizontal deformation of the streamlines. The vertical deformation is not now subject to an exponential decrease upwards; the amplitude remains nearly constant but there is a retardation of phase with increasing height.

For $\kappa < l_f$, (Fig. 1(c)), the order of the wavelength is $> 200\pi$ km. and the scale is such that geostrophic forces predominate in determining the deformation which is therefore almost entirely horizontal. Queney studied this further and found it necessary to introduce on this scale the latitudinal variation of the Coriolis parameter but he has not published a full acount of this study.

(c) Generalisation of Queney's results

As the differential equation of the wave is linear we are at liberty to superpose solutions to achieve a practical result. For a realistic two-dimensional mountain ridge, Queney used the profile presented by the Fourier integral

$$\zeta = ab\int_0^\infty e^{i\kappa x - \kappa a}\, d\kappa$$

$$= ab\int_0^\infty e^{-\kappa a}\cos \kappa x . d\kappa$$

$$= a^2 b/(a^2+x^2) \quad \text{- - - - - - - - - - (6)}$$

This is a synthesis of sinusoidal ground corrugations for all values of the wave number κ, with the exponential factor $e^{-\kappa a}$ applied to each of them. The result is a single corrugation of bell-shaped cross-section. We see that its height is b and the half width is a (the half width is the value of x, for which the height is half the maximum). We must ensure that $b \ll a$ otherwise the assumption that the disturbance quantities are small relative to their undisturbed values would be invalidated.

The displacement of the streamlines for flow over the ridge represented by Eq.(6) will be given by

$$\zeta = ab\sqrt{\frac{l_0}{l}}\int_0^\infty e^{i\left(\kappa x + \kappa\sqrt{\frac{l_s^2-\kappa^2}{\kappa^2-l_f^2}}\, z\right) - \kappa a}\, d\kappa \quad \text{- - - - (7)}$$

The problem therefore reduces to the evaluation of this integral for the various cases with which it is desired to deal. The integral cannot be evaluated exactly in the general case but approximate solutions can be obtained. Queney found that the nature of the result depended very much on

/the

the scale of the ground deformation, represented by a , in relation to the critical values $1/\ell_s$ and $1/\ell_f$. The most significant of his results can be summarised as follows.

(i) $a \ll 1/\ell_s$.

This corresponds to the flow over a very small hill of width up to about 100 metres. The effects of stability and the earth's rotation (represented by ℓ_s and ℓ_f in the integral) both prove to be negligible and the streamlines are given by

$$\zeta = ab\sqrt{\ell_0/\ell}\int_0^\infty e^{ikx - kz - ka}\,dk$$

$$= ab\sqrt{\ell_0/\ell}\;\frac{a+z}{(a+z)^2+x^2} \qquad \text{------(8)}$$

The streamlines are given in Fig. 2 and it is seen that there is a single symmetrical wave over the hill. The displacement decreases both vertically and horizontally away from the hill and there is no train of lee waves.

(ii) $1/\ell_s \ll a \ll 1/\ell_f$

In this case the width of the ground obstruction is of the order of a few km. Queney found that on this scale the effect of the earth's rotation was still negligible but that the stability became important. The form of the integral applicable in this case is

$$\zeta = ab\sqrt{\ell_0/\ell}\int_0^\infty e^{ikx + i\ell_s z - ka}\,dk$$

$$= ab\sqrt{\ell_0/\ell}\;\frac{a\cos\ell_s z - x\sin\ell_s z}{a^2+x^2} \qquad \text{---(9)}$$

The streamlines are illustrated in Fig. 3. There is a single wave above the hill having sinusoidal variation in the vertical, but no lee wave train. The important characteristic is that with the entry of stability into the result (represented by ℓ_s) the airstream is disturbed throughout its depth as distinct from the last case in which the disturbance decreased rapidly upwards. Geostrophic forces play no part and, accordingly, there is no horizontal deformation of the streamlines.

/(iii)

(iii) $1/\ell_s \ll a \doteq 1/\ell_f$

This is the case of a still broader ground obstruction whose width is of the order of 100 km. Both stability and geostrophic forces are significant on this scale and we have

$$\zeta \doteq ab\sqrt{\ell_0/\ell}\int_0^\infty e^{i\left(\kappa x + \kappa\sqrt{\frac{\ell_s^2-\kappa^2}{\kappa^2-\ell_f^2}}\,z\right)-\kappa a}\,d\kappa \quad \cdots (10)$$

After a transformation of co-ordinates Queney derived an approximate value for this integral in terms of Bessel functions and the associated Struve functions. His streamlines for this case are given in Fig. 4. There is a lee wave system with horizontal as well as vertical displacement of the streamlines. The horizontal disturbance is much larger than that in the vertical and the horizontal wavelength is of the order of several hundred km. (actually the horizontal wave length tends to $2\pi/\ell_f$ for large x .) As regards the vertical deformation of the streamlines, there is a retardation of phase with height, the vertical wavelength being $2\pi/\ell_s$, i.e. dependent on stability. The lee waves increase in amplitude upwards and persist further downstream with greater height. In many respects, therefore, these "Queney" waves are unlike those responsible for ordinary lenticular clouds and exploited by glider pilots. We shall return to this later.

(iv) Queney also studied the case of a westerly airstream blowing across a very broad mountain range having a width of the order of 1000 km. i.e. the case of $a \gg 1/\ell_f$. On this scale the variation of Coriolis parameter with latitude also figures in the result and a system of stationary lee waves, mainly horizontal, was obtained, the wavelength being several thousand km. These waves have the nature of stationary Rossby waves and although of great interest from the synoptic viewpoint need not be discussed further in this report.

(d) The work of Lyra

This author published theoretical studies in 1940 and 1943 which at that time appeared highly significant and suggestive (13), (14). Lyra, too, employed the perturbation method but for the profile of his mountain ridge he used a narrow rectangular section developed from the Fourier integral $\int_0^\infty \cos \kappa x \, d\kappa$; the resulting profile has steep vertical sides. In his solutions he obtained a type of lee wave system in which the waves increase in amplitude with height and decrease downstream. Queney (and Scorer) used the mountain profile $z = a^2 b/(a^2+x^2)$ which with $b \ll a$ has a smooth shallow section with gently sloping and symmetrical sides. It is clear from the work of both latter writers that Lyra's waves are dependent upon the steepness of the mountain slopes in the profile used by him and that they disappear when a smooth mountain replaces the rectangular one. Furthermore, it is unrealistic to adopt a rectangular mountain section as a model for study whilst at the same time assuming laminar flow. Thus although of considerable importance when published, Lyra's theory is largely superseded by the work of Queney and Scorer and will not be discussed further here.

/(e)

(e) The work of Scorer

Scorer recognised the inadequacies of the solutions obtained by Queney and Lyra. In both, the waves are almost negligible at low levels and die away rapidly downstream. In addition, Lyra's waves are somewhat artificial in that they depend on the choice of an unrealistic ground profile, whilst Queney's waves given in Fig. 4 are of very long wavelength and are mainly a horizontal oscillation about the geostrophic wind. Scorer realised that the observational evidence of mountain wave clouds and the experiences of glider pilots etc. suggested lee waves of a different character, viz. waves which increased in amplitude upwards to some level and then decreased to die away at great height, which had a wave length of a few km. and which in the absence of friction would continue indefinitely downstream over level ground to the lee of the obstacle.

Scorer (15), (16) considered frictionless, steady, laminar and isentropic flow but set out to provide for variations of lapse rate and wind shear in the vertical. He derives a second order partial differential equation for the stream function of the disturbance and simplifies this on similar grounds to Queney. The earth's rotation is then neglected, as his subsequent study relates to small wavelengths (up to 50 km. say). The simplified equation is:

$$\frac{\partial^2\psi}{\partial z^2} - \left(\frac{g}{c^2} + \beta\right)\frac{\partial\psi}{\partial z} + \left(\frac{g\beta}{U^2} - \frac{1}{U}\frac{\partial^2 U}{\partial z^2} - K^2\right)\psi = 0 \quad \cdots (11)$$

The notation is as used for Eq. (1) applicable to Queney's work and we see, in fact, that the equation is the same except that the earth's rotation term in f does not appear, owing to the smaller scale of the problem to be studied, and there is the additional term in $\partial^2 U/\partial z^2$ representing the vertical rate of change of shear (U now varies with z).

Scorer neglects the second term in $\partial\psi/\partial z$ on the grounds that its effect is small and quantitative only. In the expressions for the displacement of the streamlines subsequently obtained, the result of this approximation is that the factor $\sqrt{\rho_0/\rho}$ which figures in Queney's results does not appear. The simplified equation used is thus

$$\frac{\partial^2\psi}{\partial z^2} + \left(\ell^2 - K^2\right)\psi = 0 \quad \cdots\cdots\cdots\cdots (12)$$

where $$\ell^2 = \frac{g\beta}{U^2} - \frac{1}{U}\frac{\partial^2 U}{\partial z^2}$$

Considering first an airstream in which ℓ^2 is constant, we may use solutions of (12) for constant coefficients. The restriction ℓ^2 = constant does not imply the uniform airstream considered by Queney and Lyra since both β and U can vary with height within this limitation. The vertical displacement will be given by

$$\mathfrak{Z} = \frac{\psi}{U} = \frac{1}{U}\left(Ae^{\mu z} + Be^{-\mu z}\right) \quad \text{.....(13)}$$

where $\mu = +\sqrt{\kappa^2 - \ell^2}$

Once again we have the two cases

(i) If μ is real, i.e. if $\kappa > \ell$, we must have A = 0 otherwise the disturbance increases indefinitely upwards.

(ii) If μ is imaginary, i.e. if $\kappa < \ell$ then

$$\mathfrak{Z} = \frac{1}{U}\left(Ae^{i\nu z} + Be^{-i\nu z}\right)$$

where $\nu = +\sqrt{\ell^2 - \kappa^2}$

We have again the difficulty that both components $e^{i\nu z}$ and $e^{-i\nu z}$ are indeterminate as $z \to \infty$. Scorer does not accept the application of the Rayleigh method of limits in this case and rejects Queney's (and Lyra's) choice of the downward component $e^{i\nu z}$. He argues instead that since the source of disturbance in the present context is at the ground, and there is no relevant energy source at high levels, the only realistic choice is the upward progressing wave. He therefore uses the solution:

$$\mathfrak{Z} = \frac{B}{U} e^{-i\nu z}$$

For an infinite sinusoidal ground profile of wavelength $2\pi/\kappa$, the streamlines of Scorer's elementary solutions are thus:

(i) $\kappa > \ell$:

$$\mathfrak{Z} = \frac{U_0}{U_z} e^{i\kappa z - \mu z}$$

$$= \frac{U_0}{U_z} e^{-\mu z} \cos \kappa x$$

where $\mu = +\sqrt{\kappa^2 - \ell^2}$

(ii) $\kappa < \ell$:

$$\mathfrak{Z} = \frac{U_0}{U_z} e^{i(\kappa x - \nu z)}$$

$$= \frac{U_0}{U_z} \cos(\kappa x - \nu z)$$

where $\nu = +\sqrt{\ell^2 - \kappa^2}$

These solutions are similar to those of Queney illustrated in Figs. 1(a) and (b) except that there is no horizontal deformation (the effect of the earth's rotation being neglected) and in the case of imaginary μ Scorer adopts the wave progressing upwards relative to the moving airstream (i.e. the loci of the crests slope forward instead of backward).

(f) Generalisation of Scorer's Results

Scorer uses the ground profile represented by the Fourier integral

$$\zeta = ab\int_0^\infty e^{ikx - ka}\,dk = a^2b/(a^2+x^2)$$

For an airstream in which l^2 is constant, the streamlines for the perpendicular flow across this ridge are given by

$$\zeta = \frac{U_0}{U_z}\,ab\int_0^\infty e^{ikx - \mu z - ka}\,dk \qquad \text{------(14)}$$

If $l = 0$, the integral can readily be evaluated to obtain

$$\zeta = \frac{ab(a+z)}{(a+z)^2 + x^2} \qquad \text{- - - - - - (15)}$$

This solution applies to an airstream having neither static stability nor shear (i.e. $\beta = 0$, U = constant) for this would be a particular case of $l = 0$. The flow is seen to be similar to that obtained by Queney for a very small obstacle and illustrated in Fig. 2. We have therefore the interesting result that the flow over a very small hillock or obstacle (up to about 100 m. in width) is dynamically similar to that of an airstream with no stability in the full scale mountain problem. As we shall see later this is the only case in which wind tunnel experiments might be carried out easily and with a good prospect of success. Scorer describes this type of flow as "aerodynamic" since gravity, stability and the earth's rotation play no part.

In order to emphasise the very important role played by stability, Scorer contrasts "aerodynamic" flow with the results obtained for an airstream with constant stability (or more precisely with constant l). The approximate value obtained for the integral of Eq. (14) becomes

$$\zeta = \frac{U_0}{U_z}\,ab\,\frac{a\cos lz + x\sin lz}{(a^2+x^2)} \qquad \text{- - - - (16)}$$

/This

This is the same as Queney's solution for $1/\ell_s \ll a \ll 1/\ell_f$ except that the sign before the term $x \sin \ell z$ is reversed. This reversal occurs because of the different interpretation of the upper boundary condition; the streamlines have the same shape but their direction is reversed. However, as stated before, the important thing to note is that with the introduction of stability there is a complete change in the flow - the airstream is now disturbed throughout its whole depth. Scorer calls this "barostromatic" flow because of the vital part played by stability, i.e. by the stratification of potential density. It is to be noted that so far no trains of lee waves have appeared in these solutions of Scorer's, and this is now examined further.

The vertical velocity $w = \frac{\partial \psi}{\partial x} = ik\psi$, and thus we may write the wave Eq. (12) as follows

$$\frac{\partial^2 w}{\partial z^2} + (\ell^2 - k^2)w = 0 \quad \text{- - - - - - - - (17)}$$

Considering the application of this equation to flow over level ground to the lee of an obstacle, we require that $w = 0$ at the ground and $w \longrightarrow 0$ as $z \rightarrow \infty$. It is clear that if these boundary conditions are imposed with constant ℓ, w will be everywhere zero and therefore we can have no lee waves with such an airstream. For realistic lee waves we must have w finite at some level(s) and also zero at $z = 0$ and tending to zero as z becomes very large. Thus $\frac{\partial^2 w}{\partial z^2}$ must change sign at least once; in fact it can be inferred that we must have ℓ^2 less in some fairly deep upper layer than in a layer below. This notion led Scorer to introduce, for purposes of theoretical study, the concept of an interface in the airstream, dividing it into two layers in each of which ℓ^2 has a constant but different value. There is then a different solution of the wave equation for each layer, corresponding to the two different values of μ, and accordingly four arbitrary constants. For the four boundary conditions required we have those at the ground and at infinite height, whilst continuity considerations at the interface supply the remainder.

If for such an airstream suffixes 1 and 2 refer to the lower and upper layers respectively and for convenience we take the interface at $z = 0$ and the ground at $z = -h$, then the vertical displacement will be given by

Upper layer: $(z > 0)$

$$\zeta_2(z) = \frac{A}{U_2} e^{-\mu_2 z} \quad \text{- - - - - - - - (18)}$$

Lower layer: $(0 > z > -h)$

$$\zeta_1(z) = \frac{A}{U_2}\left(\cosh \mu_1 z - \frac{\mu_2}{\mu_1} \sinh \mu_1 z\right) \quad \text{- - (19)}$$

/Here

Here the boundary conditions at the interface and at infinite height have already been introduced whilst A awaits the ground boundary condition. For the flow over level ground to the lee of the (so far) unspecified disturbance of wave number K, waves may only occur if $\zeta_1(-h) = 0$, i.e. if

$$\mu_1 \cosh \mu_1 h + \mu_2 \sinh \mu_1 h = 0 \quad - - - \quad -(20)$$

As $\mu_1 = \sqrt{K^2 - l_1^2}$ and $\mu_2 = \sqrt{K^2 - l_2^2}$, Scorer regards Eq. (20) as an equation in K determining the wave number of those waves which could persist in the airstream over level ground when once initiated by a disturbance upwind. He shows that Eq.(20) only has solutions if $l_1^2 - l_2^2 > \pi^2/4h^2$ and in that event there may be more than one value of K. However, it is also shown that the lee waves corresponding to the smallest root, denoted by K_0, will be amplified to a much greater extent than the others which in practice may be ignored.

Allocating the value of A for flow over the sinusoidal ground profile e^{iKx} (real part) the streamlines in the lower layer become

$$\zeta(z) \rightleftharpoons \frac{e^{iKx} f_z(K)}{f_{-h}(K)}$$

where $f_z(K) = \frac{1}{U_z}\left(\cosh \mu_1 z - \frac{\mu_2}{\mu_1} \sinh \mu_1 z\right)$

Generalising, we have that the streamlines for the flow over the ridge represented by

$$\zeta(-h) \rightleftharpoons ab \int_0^\infty e^{iKx - Ka}\, dK = a^2 b/(a^2 + x^2)$$

will be given by

$$\zeta(z) \rightleftharpoons ab \int_0^\infty \frac{f_z(K)}{f_{-h}(K)} e^{iKx - Ka}\, dK$$

/This

This may be evaluated by contour integration but as $f_{-h}(K_0) = 0$, the integrand is infinite for $K = K_0$. Scorer invokes the stationary phase principle to overcome this difficulty and derives as a close approximation for the streamlines in the lower layer:

$$\zeta(z) \rightleftharpoons ab \frac{f_2(0)}{f_{-h}(0)} \frac{(a+ix)}{(a^2+x^2)} + 2i\pi ab \frac{f_2(K_0)}{\frac{\partial f_{-h}(K_0)}{\partial K_0}} \quad \text{for } x>0 \quad \text{--- (21)}$$

$$\zeta(z) \rightleftharpoons ab \frac{f_2(0)}{f_{-h}(0)} \frac{(a+ix)}{(a^2+x^2)} \quad \text{for } x<0 \quad \text{--- (22)}$$

In the upper layer the lee waves, represented by the second term in the expression for $x>0$, die away exponentially being proportional to $e^{-\mu_2 z}$

Scorer has exploited this result for a simple two-layer model to calculate the streamlines in selected cases. One of his examples is reproduced at Fig. 5. Since $l^2 = g/\beta/U^2 - \frac{1}{U}\frac{\partial^2 U}{\partial z^2}$, there is considerable latitude in the choice of the wind and temperature profiles whilst still ensuring that l^2 is constant within each of the two layers. In fact in the example illustrated, there are effectively three layers but the value of l^2 in two of them is the same. The characteristics of the wind and temperature profiles used by Scorer for this example are shown on the left of Fig. 5; they are similar to those commonly found in nature when waves are observed, viz. marked forward wind shear through a stable layer at fairly low levels. The lee waves predicted by the theory also accord well with what is commonly observed. By avoiding the artificiality of the uniform airstream case studied by Lyra and Queney, and dealing with more realistic model airstreams, Scorer has revealed the significance of variations of lapse rate and wind speed in the development of lee waves.

Multilayer models, with several different values of l^2 could in theory be set up and examined in similar fashion. However, the mathematics would become inordinately complex and no new general principle could emerge. In practice where l^2 may vary continuously it would remain true that for lee waves to occur l^2 must assume lower values through some fairly deep upper layer than in some layer below (but see qualifying comments below re inversions).

We have led up to this result in some detail because it is of fundamental importance and it seems likely that it may prove useful for practical forecasting. Scorer has, however, deduced a number of other conclusions which it is now proposed to summarise without giving the derivation in full detail.

Two-Layer Models

(i) In the case of the simple two-layer model, Eq. (20), which determines the wavelength, only has solutions if $l_1^2 - l_2^2 > \pi^2/4h^2$. Thus the magnitude of the decrease in l^2 must exceed a limit depending on the depth of the lower layer.

If $l_1^2 - l_2^2 > \frac{9\pi^2}{4h^2}$ there will be one nodal surface within the lower layer, where the wave has zero amplitude and the phases above and below are opposite.

If $l_1^2 - l_2^2 > 25\pi^2/4h^2$ there will be two nodal surfaces, etc.

(ii) Distinguishing for a moment between the disturbance over the hill itself (the so-called hill-lift) and the lee wave train, if any, the hill-lift disturbance will be greater and will extend to greater heights if l^2 decreases upwards i.e. the condition for a large disturbance over the hill itself is the same as for lee waves.

(iii) We have noted that the amplitude of the lee waves increases upwards to some level above which it decreases to die out in the upper levels. For a given simple two-layer model the level of maximum amplitude can be obtained from Eq. (21) but in the general case we may say that the maximum amplitude will occur roughly where l^2 is greatest.

Adiabatic Layers

(iv) In the atmosphere adiabatic layers in the lower levels are common. In such layers $l^2 = 0$ (neglecting shear) and it might appear that Scorer's condition for lee waves could not be satisfied. However, the condition for lee waves is that l^2 should be greater in some lower layer than in the layers above; the largest values of l^2 do not necessarily have to occur in the surface layer. Scorer has studied this further and finds that waves are still possible with an adiabatic ground layer, provided his criterion is satisfied above, but the magnitude of the disturbance aloft will then be less than with stable stratification down to the surface. He has in fact published the streamlines calculated for a three-layer model in which the bottom layer is adiabatic (Fig. 7 in ref. (16)) and obtained lee waves in such a case.

(v) If there is an adiabatic layer (or more generally a layer in which $l^2 = 0$) in middle levels, and one starts from the assumption that steady laminar flow with lee waves is possible, it is found that the boundary conditions at the top and bottom of the adiabatic layer are related. Since these conditions are necessarily independent it is concluded that the kind of flow assumed cannot occur, i.e. that steady laminar flow over a mountain must break down if there is an adiabatic layer in middle levels. Perturbation theory cannot assist in determining the type of flow in such a case.

/Inversions

Inversions

(vi) The development of stationary lee waves on flowing water is closely connected with the large density discontinuity at the surface. Several writers, notably Küttner, have recognised that an inversion in the atmosphere might possibly act in the same way and thus have a bearing on the occurrence of atmospheric lee waves. However, the work of Queney and Scorer indicates that one need not introduce an inversion in the model airstream considered in order to obtain lee waves. Scorer, in fact, has been at great pains to emphasise in many papers that an inversion is not a dynamical requirement for lee waves. It is true that an inversion is commonly found in airstreams in which waves are observed, but as has been pointed out an inversion would commonly be associated with a distribution of humidity favouring wave clouds, i.e. conditions in which any waves become visibly revealed. Scorer has now agreed, however, (Weather, July 1953) that although not a dynamical necessity for waves, an inversion could make them possible when his criterion regarding the variation of l^2 with height did not appear to be otherwise fulfilled. An ideal inversion would be a layer of no depth and infinite l^2, but in practice we can regard an inversion as a very shallow layer through which the value of l^2 is very large. The occurrence of waves in airstreams containing inversions is therefore quite consistent with the general criteria regarding the variation of l^2 with height. However, further theoretical study of the role played by inversions (and by shallow but very stable layers) would probably be profitable.

Diurnal Variation

(vii) In the lower layers of the atmosphere, the lapse rate, wind speed and wind shear (and therefore the value of l^2) are subject to large diurnal variations especially near sunrise and sunset.

Generally speaking the net result is a reduction in the value of l^2 after sunrise and an increase around dusk. In theory this would mean an increase after sunrise in the wavelength of any waves until perhaps the waves ceased owing to infringement of the condition $l_1^2 - l_2^2 > \pi^2/4h^2$, whilst around dusk the waves might reform if and when the condition was again satisfied. Both these effects have been observed and reported by other writers (2), (17). It is appropriate to mention here that on similar grounds, one would expect a greater frequency of wave effects in winter than in summer; this too is in accord with observation.

Oblique Flow and other Mountain Shapes

(viii) So far discussion has been limited to the airflow perpendicularly across an infinite symmetrical ridge. This lends itself to theoretical study because the problem is then essentially only two-dimensional. In theory one can deal with more complicated two-dimensional profiles by superposing the solutions for different ridges. In this way the streamlines for the flow across an asymmetrical ridge and various plateaux have been obtained.

For oblique flow across a ridge, the component along the ridge remains undisturbed whilst the component across the ridge is disturbed as in the case of perpendicular flow. Scorer has shown (16) that if

/the

the undisturbed stream contains a component of shear parallel to the ridge, the total shear in the disturbed stream may assume almost any direction. This conclusion has a bearing on the formation of billow clouds and also may be significant in connection with the navigation of aircraft over mountain ranges.

Theoretical study of the flow over an isolated hill (as distinct from an infinite ridge) is complicated since the problem becomes fully three-dimensional. One can exploit the result for oblique flow over a ridge, by synthesising the flow patterns for an infinity of ridges whose axes are inclined in all horizontal directions through a point. In this way Scorer was able to obtain the distribution of vertical displacement for the flow over a single hill (18) and his results are consistent with the bow-shaped orographic cloud which has been observed in such a case and discussed by Abe (1). As regards the development of waves to the lee of a single hill, the general conclusions are that any waves will be of smaller amplitude than those to the lee of an infinite ridge of similar section and that the waves will decrease in amplitude rapidly downstream. This is in contrast to the infinite ridge case when the lee waves (in the absence of viscosity) persist indefinitely downstream. For a ridge of finite length lying across the airstream the results are intermediate, viz. the persistence of any lee waves downstream is greater the greater the length of the ridge.

A Layer of Calm Air

(ix) If an airstream has a layer of calm air between some levels the difficulties about the flow at great heights, and in particular about the boundary condition at infinite height, disappear. Scorer studied such a case and found that lee waves, confined to the airstream below the calm layer, were possible provided certain conditions were satisfied. This case appeared to be of academic interest only, but recently a case of lee waves has been reported (19) in circumstances which were possibly of this type.

(g) The Validity and Limitations of the Results of Perturbation Theory

In order to assess the practical value of the theoretical results deduced from the applications of the perturbation method, it is necessary to review the initial assumptions. The main assumptions involved are commented on below.

(i) Viscosity. The effect of the assumption of frictionless flow was examined by Queney, who concluded that viscosity would merely exert a damping effect on the disturbances. The damping effect would be small and would not alter the results qualitatively.

(ii) Isentropic Flow. The assumption that the potential temperature is constant along the streamlines involves neglect of radiation and condensation effects. Radiative effects would in general be too slow to affect the results on the scale considered by Scorer, but on the synoptic scale (e.g. for the long waves of the stationary Rossby type) they would probably be important. However, the latter question has not been fully studied and is irrelevant in the present report.

As regards condensation, the results could easily be applied to a completely cloudy airstream by using wet bulb potential temperature in the coefficient of stability β. One could similarly allow for an 8/8 layer of cloud but not readily for broken clouds. In the case of an airstream containing, say, well broken cumulus, most of the atmosphere (i.e. that between the clouds) would be stably

/stratified

stratified and the conclusions from perturbation theory would be closely applicable. This means that convection is not necessarily inconsistent with the simultaneous occurrence of orographic waves, although it must at least complicate the flow to some extent by mutilating the laminar flow.

(iii) Steady Laminar Flow. The restriction to steady laminar flow is serious because there is ample evidence that the flow over mountains is commonly otherwise. No mathematical treatment of unsteady or non-laminar flow has yet been put forward and it is unlikely that much progress could be made by such a theoretical study until some completely new and superior mathematical approach is forthcoming. In the meantime, we must be content with what has been learnt of the flow in such cases from observational evidence. Here the work of Förchtgott, although in many respects inconclusive, is valuable and illuminating and it is probable that extended field studies on similar lines would be profitable. (Förchtgott's work is discussed in Part IV of this report.)

(iv) Displacements Small. The displacements, and indeed all the disturbance quantities, are taken to be small compared with the corresponding undisturbed values. This enables the equations to be linearised since the products and squares of the disturbance quantities are then justifiably neglected. The condition is satisfactorily fulfilled if the height of the mountain is small compared with its width. It is reasonable to accept the results as qualitatively correct (and very nearly quantitatively also) provided the mountain height is no more than about 10% of the width, although Queney claims rough validity if the ratio "height to width" is up to 1/4 or even 1/2 in special cases. We must bear in mind, however, that with increasing steepness of the mountain ridge, separation of the flow from the surface becomes increasingly probable. This would be manifest in the form of lee eddies or some other form of motion and would constitute, in effect, violation of the assumption of laminar flow.

(v) The Earth's Rotation. For Queney's waves, the earth's rotation provides one of the essential restoring forces, the scale of the phenomena considered being large enough for this to be possible. For Scorer's lee waves, the scale is too small for the earth's rotation to have any significant effect since he confines his study to cases when the mountain ridge and the régime considered are traversed by the airstream in an hour or two at the most. With this restriction, neglect of the earth's rotation in Scorer's examples is quite justified, but his results would not be applicable to large scale phenomena.

PART III - MODEL EXPERIMENTS

The hope that model experiments could be used to advance our understanding of the difficult problem of mountain airflow has been cherished by many writers on the subject in the past and a number of attempts has been made to exploit model techniques. These attempts have met with little or only partial success. In some cases no consideration at all was given to the need for achieving dynamical similarity between the model and the full scale problem, whilst in other cases the only precaution taken was to obtain a reasonable Reynolds number in the model on the assumption that molecular viscosity in the model would correspond to eddy viscosity on the full scale. In defence of these past attempts, however, it must be mentioned that the

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theoretical studies described in the last section were not available at the time most of the model experiments were carried out; furthermore no general study of the conditions for dynamical similarity of atmospheric motions, such as that published recently by Batchelor (30), had then been made. It is probable therefore that the very great importance of, and the criteria for, dynamical similarity were not appreciated by model experimenters in the past.

The differential equation used by Scorer to determine the wave motion, in cases when the scale of the problem permits the earth's rotation to be neglected, is:

$$\frac{\partial^2\psi}{\partial z^2} + \left(\frac{g\beta}{U^2} - \frac{1}{U}\frac{\partial^2 U}{\partial z^2} - K^2\right)\psi = 0$$

(Part II, Eq. (12), notation as before).

Here $g\beta/U^2$ represents the effect of stability or the stratification of potential density and $\frac{1}{U}\frac{\partial^2 U}{\partial z^2}$ represents the effect of the vertical rate of change of shear. K is the wave number having the units (length)$^{-1}$, and figures instead of a representative length. Scorer argues that if a model experiment is to reveal accurately the motion in the full scale problem, the conditions in the model experiment must be so arranged that the relationship between the terms of the equation is the same both for model and full scale. That is, the non-dimensional number:

$$S = \frac{1}{K^2}\left(\frac{g\beta}{U^2} - \frac{1}{U}\frac{\partial^2 U}{\partial z^2}\right) \quad \text{.....} \quad (23)$$

must have the same value on both scales. We must bear in mind, however, that Eq. (12) is not exact, several assumptions and approximations being involved in its derivation. It is not, therefore, immediately apparent that the conditions for dynamical similarity can be deduced in such a manner from the approximate wave equation. The conditions so obtained depend on the assumptions and approximations being justified on the model, as well as on the full scale, and a separate examination of the relative magnitude of the terms of the full equation as applicable to the model problem would seem to be first necessary. Alternatively, one could avoid this difficulty by deducing the conditions from the basic equations or motion, i.e. before linearization and approximation. For inviscid, isentropic flow, such that Coriolis forces can be neglected, Batchelor (30) deduced in this way the criteria to be equality of the non-dimensional numbers:

$$\frac{gL}{U^2}, \quad \frac{p}{\rho U^2}, \text{ and } \gamma$$

where L is a representative length (equivalent to $1/K$ in Eq. (23)). He found also that if the flow field is assumed to be such that the departures of pressure and density from values in an equivalent isentropic atmosphere are small and that the vertical scale of the motion is small, the second and third non-dimensional numbers disappeared from the dimensionless

/equations

equations of motion. He was left with a single condition, viz. equality of a special form of Richardson number which proves to be equivalent to the "Scorer number" $S = g\beta L^2/U^2$. His special assumptions regarding the flow field are satisfied in the atmosphere if the perturbation is small, so that it does in fact appear that Scorer's condition for similarity is adequate to a degree. The important limitations are that equality of the "Scorer number" would not justify conclusions from model experiments regarding turbulent flows or flows in which the perturbation was not a first order quantity. The more rigorous conditions of Batchelor would need to be satisfied for this higher degree of similarity.

Assuming that the limited degree of similarity achieved by equating the "Scorer number" were acceptable for a particular experiment, the model flow conditions can be calculated for a typical case. For a model 3×10^{-5} times full size, with $U = 10^3$ cm./sec. and $\beta = 2 \times 10^{-7}$ cm.$^{-1}$ (a lapse rate of about 4°C/km.) on the full scale, then even with the very large temperature gradient of 1°C/cm. in the model, the wind tunnel flow speed would have to be as low as 4 cm./sec. For faster flow speeds very much greater temperature gradients would be required. It is clear that Scorer regarded it as impracticable to obtain such experimental control and abandoned pursuit of the possibility of wind tunnel experiments.

It is interesting to consider for which full scale cases wind tunnel experiments could legitimately be regarded as valid, if no control at all is exercised over the temperature gradient in the model. If in the wind tunnel the temperature and flow speed are constant, then the "Scorer number" for the model is zero. Thus the experiment should reproduce the full scale flow only in the special case when $l^2 = 0$ on the full scale. Such experiments might, therefore, be of limited value if one wished to investigate the flow over hills in, say, Northern India in the pre-monsoon period when the lapse rate is nearly dry adiabatic up to 10,000 feet or more, i.e. a case when the static stability is zero or very small for a considerable depth. In addition, and this might be more useful, the experiments could be applied to examine the flow of a completely cloudy airstream over a mountain for then the static stability (and thus l^2) would be approximately zero in the full scale case as well as in the model. The type of case in mind for this is one in which the airstream contains 8/8 nimbostratus and has a wet adiabatic lapse rate throughout a great depth. Apart from the special full scale case in which $l^2 = 0$, we may infer from the theory of the last section that a wind tunnel experiment without control of temperature would reproduce the flow over a small obstacle or hill up to about 100 m. in width since, as we saw, the full scale flow in such a case is practically independent of the stability effect, i.e. of the variation of l^2.

If we wished to devise model experiments to investigate larger scale phenomena in which the earth's rotation is concerned, e.g. Queney waves, it would be necessary for the non-dimensional number

$$Q = \frac{f^2}{k^2 U^2} \qquad \text{(24)}$$

to be equated in both model and full scale. This condition is effectively the same as equality of the Rossby number and must be satisfied in addition to equality of the Scorer number. The condition follows from consideration of Eq. (1) and it would ensure the proper representation of geostrophic forces in the model experiment. In a typical case, the condition would require rotation of the model at about 1 revolution per 2/3 minute to

/simulate

simulate the earth's rotation. It seems unnecessary to pursue this further here since mountain airflow on the large scale implied is not of direct concern to aviation.

In connection with the Gibraltar investigation of 1929/30 (20), Field and Warden supplemented their field work by wind tunnel experiments on a 1 in 5,000 model of the Rock. As they made no arrangements to simulate stability, the flow obtained in the model should have corresponded to the flow in the full scale case of an airstream with zero static stability. In spite of this, what they described as remarkable agreement between the flow in the experiment and the flow on the site was obtained. We must remember, however, that the flow on the site was in no sense fully mapped out; it was only possible to determine some of the characteristics of the flow to a limited extent from the programme of balloon work. Furthermore they were dealing with a constantly changing turbulent flow pattern which made specific comparison between the model experiment and the site work much more difficult than would have been the case had the flow been steady and laminar.

Abe studied the mountain clouds around Mount Fuji in Japan from 1920 onwards and from time to time supplemented his site study with wind tunnel experiments (21). He used a 1 in 50,000 model and selected a tunnel flow speed so as to equate the Reynolds number to that applicable on the full scale; this was, of course, on the usual assumption that molecular viscosity in the model corresponded to eddy viscosity on the full scale. The tunnel flow speeds required for this were of the order of 2 m.p.h. In an attempt to achieve greater realism, Abe arranged wind shear and a temperature gradient in the model experiments. For wind shear, he introduced an extra blower or used partial interruption of the flow, and for a temperature gradient he introduced dry ice smoke at the bottom of the tunnel. He also reproduced a change of wind direction with height by introducing blowers at the side of the tunnel. So far as is known, this has been the only attempt to simulate shear and stability in wind tunnel experiments. Unfortunately Abe did not arrange these effects quantitatively to ensure that strict dynamical similarity was achieved, and we must assume that if there was complete similarity it was accidental. He photographed the model flow and found approximate qualitative agreement with some properties of the flow on the site, insofar as he had deduced them from observations of the mountain clouds. In particular in both cases many vortices were observed with vertical, horizontal or inclined axes. Abe's work tells us a good deal about the characteristics of the flow over and around Mount Fuji; it assists very little towards a more complete understanding of the problem of mountain airflow in general.

Rouse (22) has discussed the potentialities of model experiments and refers, without giving details, to the value of experiments on large models of mountains in hangars using blowers or airscrews to provide the flow. We have seen that a model experiment in a wind tunnel without control of stability will with certain restrictions give dynamical similarity with cases of the flow over obstacles and small hills up to about 100 m. in width (and in addition with a full scale mountain problem in the special case of $l^2 = 0$). To proceed, therefore, from the wind tunnel scale to the scale of hangar experiments does not avoid the awkward problem of controlling the temperature gradient to simulate stability. Some advantage might derive from the increase of scale, however, insofar as much smaller temperature gradients would be required on the hangar scale than in the case of the wind tunnel.

So far, we have not discussed viscosity. If viscosity is introduced into the equations of motion by means of the usual terms, it is found that for dynamical similarity the Reynolds number must be equated on both scales

/in

in addition to the other dimensionless numbers already discussed. It has usually been suggested that if this equality is achieved by using molecular viscosity in the model and eddy viscosity in the full scale, the simulation of this feature will be satisfactory. On the face of it, this idea seems of dubious validity and although doubt has been expressed by some writers, it does not appear to have been thoroughly examined in the literature. Possibly the similarity hypotheses and other ideas of Kolmogoroff and Batchelor recently discussed by the latter (31) would be of value here.

In the course of a study of the flow of stratified liquids, Long has commenced experiments using a three-layer system of immiscible fluids in a long glass-sided tank. Long has recently given a preliminary account (23) of his first experimental results. In this case the obstacle, which filled the space between the front and back glass walls, was steadily drawn at various speeds along the bottom of the tank by mechanical means. The deformations of the three layers of fluid were studied and photographed. Long applied a rough dimensional analysis to the obvious parameters of his model arrangement and considered that his experimental conditions roughly reproduced the effect of stability and in fact suggested that the two interfaces might approximately correspond to an inversion at mid-tropospheric levels and the tropopause. The flow obtained in the three-layer system exhibited broadly some of the characteristics of the "Bishop" wave. In some of Long's photographs there is some sign of the fluid ascending towards the obstacle at low levels and descending at higher levels above the same position. This is interesting since the streamlines calculated by perturbation theory for many atmospheric cases show the same feature and the validity of this aspect of the theoretical results has not in the past been readily accepted. The use of layers of liquid of different densities for the simulation of stability appears to have much to recommend it as, presumably, the adjustment of densities in such an arrangement poses a far simpler laboratory problem than the production of a very large temperature gradient in a wind tunnel. Admittedly the compressibility of the atmosphere is not reproduced, but as was seen from Batchelor's work this can be ignored if first order perturbations only are to be studied. Even in a wind tunnel dynamical similarity for large amplitude perturbations requires what seem to be impossibly rigorous conditions, so that in this respect the stratified liquid model is on at least an equal footing. Long was quite conscious of the limitations of his experiment and in future work intends to simulate more closely a continuous density gradient by using a large number of layers of liquid and also to use a scale model of the Sierra range instead of an arbitrary obstacle. Long's future results are awaited with interest.

PART IV - THE WORK OF FÖRCHTGOTT

Förchtgott has carried out intensive investigations of the airflow over ridges during the post war years (24), (25). He is, amongst other things, a gliding instructor who has specialised in soaring in mountainous areas and in view of his wide experience his ideas on the subject must command respect. His work has comprised a series of field studies at various mountain locations in Bohemia using light aircraft and groups of gliders fitted with instruments. The flights were supplemented by the study of mountain clouds (both visually and by time lapse photography), of smoke and of the flight of birds. In all, during the period 1946-1950, he gathered about 35 different sets of observations spread over 5 different mountain locations. By studying the observational data gathered and drawing on his own considerable gliding experience, Förchtgott has built up a fairly comprehensive theory which is presumably

/consistent

consistent with both the data and his experience. His classification of mountain airflow into four main types is attractive and plausible but is deduced mainly by a process of subjective interpretation. It is not clear to what extent we can regard his conclusions as being of general validity.

Förchtgott has been severely critical of the conclusions deduced from perturbation theory on the grounds that by neglecting the turbulent layer and treating the flow as wholly laminar and steady, theorists have ignored a vital feature of mountain airflow. However, it must be stated that the significance of the restriction to steady laminar flow has been most fully acknowledged in all theoretical treatments, and in point of fact there is little or no conflict between Förchtgott's results and those of perturbation theory.

Förchtgott found that no organised flow patterns could be discerned unless the airstream had stable or neutral stratification, the wind was roughly normal to the mountain ridge and the wind direction varied little with height. Accordingly his studies were confined to situations when these conditions were satisfied. He found that mountain ridges with steep lee slopes were most favourable for lee wave effects and that the topography of the upwind slope had no evident importance. Apart from these provisions he concluded that the type of flow depended on the speed of the wind as compared with the height of the ridge above the lee plain and the variation of wind with height. His classification on this basis into four main types of flow is given in Fig. 6, the wind profile which he found to be appropriate to each flow type being indicated at the left of each diagram. With the exception of the first type (Fig. 6(a)) the flow patterns are non-laminar (and possibly unsteady) and we must presume, therefore, that Förchtgott's streamlines are merely symbolic. The flow types are now briefly commented upon.

(a) Undisturbed Streaming

This occurs with very light winds; each streamline executes a single shallow wave and the vertical displacement of the streamlines dies out rapidly with height.

(b) Standing Eddy Streaming

Slightly stronger winds are required for this type. The surface streamline separates from the ground and there is a stationary lee eddy with approximately horizontal axis. The crest in the laminar streaming above the eddy is displaced downwind from the mountain, and we may regard the effective shape of the mountain as being changed by the presence of the eddy. Reverse flow up the lee slope is implied in this type of flow and has been observed frequently in mountain areas. Although not specifically stated by Förchtgott, it is obvious that zero static stability would favour the formation of the lee eddy.

(c) Wave Streaming

With still stronger winds, the stationary lee eddy disappears and a system of lee waves develops in association with a stationary vortex system having both clockwise and counter-clockwise rotations in two horizontal rows. Turbulence is found in the vicinity of the vortices. The waves die out both horizontally and vertically away from the ridge. Thus Förchtgott does not find in practice steady laminar wave streaming down to the surface on the lee side. He always finds some form of turbulent wake to the lee of the ridge even when there are smooth waves above. However the evidence for stationary closed vortices does not seem conclusive and we must remember that an impression of rotation of the bar clouds, or so called "rotors", would be provided by the wind

/shear

shear in the wave system. The turbulence too could be accounted for by shear, which in theory would be intensified in the crests and troughs of the waves.

(d) Rotor Streaming

With very strong winds extending through a restricted vertical depth comparable with the height of the mountain (see wind profile in Fig. 6(d) and (e)), the double vortex system becomes intensified and the spacing closes up towards the mountain. There is in this case no significant wave system above the vortices. Flight investigation of this type of flow was found very difficult owing to heavy turbulence, expecially close to the lee slope. In discussing the conditions for this type of flow, Förchtgott introduces the term "streaming layer" and speaks of critical values of its depth. The streaming layer is not defined, but he appears to have in mind the layer in which there is a definite component across the ridge either increasing or remaining roughly constant with height. A strong component of reverse shear, reducing the component across the ridge to zero or small values at some level, is regarded as providing a streaming layer of limited depth. Förchtgott claims that wave streaming breaks down into rotor streaming when the depth of the streaming layer falls to about $1\frac{1}{2}$ times the height of the ridge. Further decrease in the depth of the streaming layer transforms the flow pattern into a completely chaotic type in which no organised fetures can be recognised.

As regards the limits of wind speed determining which of the types of flow (a), (b), (c) will appear in any given case, Förchtgott's observations indicated that these would depend upon the height of the ridge above the lee ground. He quotes, for example, that for a ridge 900 metres high, a wind of 12 m./sec. is necessary for wave streaming. If the obstacle is a solitary hill, rather than a very long ridge, a much stronger wind speed is necessary for a given type of flow than in the case of a long ridge of a similar height.

Förchtgott has assembled his various deductions in the form of a diagram from which, it is claimed, one could decide which type of flow to expect in any given case. The variables are the height of the hill, the wind speed and the depth of the streaming layer, but there is no provision for the length of the ridge. Something of this sort would be extremely valuable for aviation forecasting but for the time being we cannot regard Förchtgott's diagram as more than a convenient summary of his own ideas. Further intensive field work would be necessary before these results could be exploited with confidence.

There is much in Förchtgott's results to confirm the predictions of perturbation theory as obtained by Scorer. The stable stratification and increase of wind with height implied for wave streaming (Fig. 6(c)) means a decrease of the parameter l^2 with height, as required by perturbation theory. The wind profile for Förchtgott's rotor streaming (Fig. 6(d) (e)) involves a large increase of l^2 with height. In such a case perturbation theory tells us that steady laminar streaming with lee waves is not possible, but is unable to indicate what type of flow occurs in its place. Förchtgott, too, does not find lee waves but has suggested the alternative from his field data.

/PART V

PART V - AVIATION APPLICATIONS

Although the state of knowledge of mountain airflow is far from being complete and satisfactory, sufficient is now known to justify some cautious and tentative applications to aviation problems, including forecasting. Experience in other fields suggests that attempts at the practical application of new work can often assist by throwing fresh light on aspects which are only partially understood, so making possible further advances. A few remarks are, therefore, given below on the possibilities, difficulties and limitations of applying current knowledge on mountain airflow to aviation problems.

(a) Aviation Forecasting

The theoretical studies reviewed in Part III included streamlines calculated for a number of special cases. It should be said at the outset that similar calculations for real cases would be totally impracticable as a normal routine in a forecast office. Firstly it would be necessary to determine a model airstream which approximated to the real airstream and was at the same time suitable for the calculations. Secondly, the computation of the streamlines would be extremely laborious and lengthy, even for the case of the flow over a simple symmetrical ridge.

The most that could be done in practical forecasting would be to exploit, mainly qualitatively, some of the general results which emerge from the perturbation theory studies and in particular to make use of Scorer's deductions regarding the significance of the variations in the vertical of his parameter l^2. There has, of course, been no extended study of reported cases of standing waves aimed at testing the validity of these deductions, but in a few reported cases where there were sufficient details to permit such an examination, e.g. in ref. (26), the l^2 profile has been calculated and the theory verified. Furthermore, if there are no large discontinuities of lapse rate, Scorer's criterion for waves reduces to a requirement for a strong forward wind shear in the air current across the ridge or mountain. This general property of airstreams containing waves is such a common feature in reported cases that there is in effect a good deal of circumstantial support for the theory. In addition it will be remembered that Förchtgott's field studies revealed the same feature.

The reports of wave effects contributed at Northolt by B.E.A. pilots permit a rough check of the theory, and this has been done in 33 cases. It was not possible in all these to select radio sonde and wind ascents representative of the area where the disturbances were found and at the same time sufficiently upwind to be unaffected by the disturbances. The profiles of l^2 cannot, therefore, be regarded as precisely applicable. However, they all showed a pronounced maximum of l^2 through some layer, the mean height of which was centred at about 800 mb., with very much smaller values above. The mean of the ratio "maximum l^2" to "value of l^2 in the upper layer" was 8·8 for the 33 cases, showing the substantial nature of the reduction of l^2 with height in these cases.

One criticism which has been voiced against Scorer's condition for waves is that most airstreams would show some decrease in l^2 with height, whereas one might well have expected some highly selective criterion to apply. In order to examine whether this criticism is

/justified,

justified, the l^2 profiles were calculated for a sample of 50 airstreams. These were chosen so that there was little change of wind direction with height, but in other respects the sample was a random one with cases both day and night and at all times of the year. As expected, it was found that some decrease upwards in the value of l^2 was common but the average value of the ratio "maximum l^2" to "value of l^2 above" for the 50 cases was only 3·3. It will be noted that this is much smaller than that obtained for the occasions when waves were reported at Northolt. In this connection, also, it will be remembered that in the case of the two-layer model airstream, it was found that the reduction in the value of l^2 from the lower to the upper layer had to attain a minimum magnitude (depending on the depth of the lower layer) for a wave solution to be possible. A decrease alone in the value of l^2 was not a sufficient condition. Unfortunately it is not possible for this further condition to be used quantitatively in practical work, since we must deal with airstreams as we find them rather than with ideal two-layer models. It can be inferred that when the profile of l^2 varies continuously, some form of condition regarding the magnitude of the decrease in l^2 is certainly necessary but this cannot yet be formulated. The most that can probably be said at present is that the greater the decrease of l^2 with height, the greater is the likelihood of waves. Here, more detailed reports than those collected in the past may assist towards a better understanding of this aspect of the problem.

In view of the above considerations it now seems that an attempt could be made to indicate to pilots when waves might be expected to operate and give rise to large areas of lift or sink for flights crossing, or passing to the lee of mountain areas. When perturbation theory indicates that steady laminar, wavelike flow to the lee of mountains cannot occur, the forecaster can at present do little more than make guarded use of the results of Forchtgott. Scorer has discussed in a recent report (27) these forecasting aspects and indicated the sort of advice which could be issued to pilots. With regard to the computation of the profile of l^2, a convenient method, in which a celluloid scale is applied to the appropriate tephigram, has now been devised by Wallington (28).

One great difficulty in bridging the gap between the theoretical results and practical forecasting arises from topographical reasons. The theory necessarily deals with simple mountain shapes, mostly infinite ridges; in practice something approaching the ideal obstacle occurs only at a few rare locations. In general the forecaster is concerned with what can only be described as irregular mountainous or hilly terrain. Over such areas on a day favourable for wave motions, the actual airflow must be the complex result of interference between many separate wave trains and for this reason alone the forecaster has little hope of saying much more than that areas of lift and sink will be encountered by a pilot. He cannot indicate the magnitude or horizontal extent of vertical currents on the basis of what is known at present.

(b) Safety Heights

One fact which emerges very clearly is that, because of the vertical motion which may occur in airstreams over and to the lee of mountains, safety heights applicable to air routes crossing mountains should provide for a greater margin of safety than is normal over level terrain. The large number of unexplained or inadequately explained aircraft accidents in mountainous country adds point to this need.

/Tentative

Tentative proposals as to what increases in safety heights should be applied have already been put forward (10), (29), but these are very much in the nature of interim suggestions and have no precise basis. This is an important operational problem but no rigorous solution seems possible on the basis of existing knowledge. Some of the more obvious considerations and difficulties are mentioned below.

(i) On a civil air route, climbing to unnecessary heights over mountains may be expensive (on account of extra fuel), sometimes unwise (due to icing liabilities) and possibly inconvenient (owing to the effect of Air Traffic Control procedures, the quadrantal height system, etc.). Therefore if airline operators are to accept a requirement for increased safety heights, it must be possible to demonstrate beyond question that the proposed increases are essential.

(ii) Consider for a moment the simple case of an airstream flowing perpendicularly across a ridge and let us assume that harmonic waves are operating in the airstream to the lee. If an aircraft is flying into wind it will be subject to harmonic fluctuations of height out of phase with the waves in the airstream. The magnitude of the aircraft's height changes will depend on the speed of the aircraft and the speed of the wind in addition to the amplitude of the atmospheric waves. If the aircraft is flying with the wind, its height changes will be in phase with the waves in the airstream but will be of smaller magnitude than when on the reciprocal course. If the aircraft is flying parallel to the ridge it could be in an area of lift or sink continuously, depending on its track relative to the wave crests. A simple formula could be deduced to give the height fluctuations of the aircraft in terms of the amplitude of the waves in the airstream, the wind speed, aircraft speed and the track being flown, and this could be used as a basis for determining safety heights. For non-laminar or unsteady flow patterns we should need to introduce the horizontal extent and magnitude of the vertical currents in place of the amplitude of the atmospheric waves.

(iii) It is clear from (ii) that any rigid solution to this problem would require the safety height to be determined for each individual flight. This would only be practicable from the point of view of the airlines if the procedure were very simple. The primary difficulty, however, is that it is not yet possible to predict the amplitude of the waves (or details of the vertical currents in non-laminar flows) except very approximately. In addition, the fact that in practice we must deal with rough irregular terrain, rather than simple ridges, makes the problem even more complicated. It therefore seems that some compromise solution is inevitable. If there were large numbers of detailed aircraft reports giving accurate values of vertical currents and their horizontal extent, it might be possible to develop on a statistical basis a procedure for fixing safety height. The information available, however, is inadequate for such an approach, and could not be obtained without aircraft flying in regions and circumstances which must now be regarded as dangerous.

(iv) There are some grounds for thinking that the worst circumstances that occur (as regards involuntary loss of height by an aircraft) are roughly equivalent to those which would be produced if the airstream exactly followed the topography. Without attempting to substantiate this here we can at least use this assumption as a basis for examining a little further the factors and difficulties involved in the problem. It is a matter of simple algebra to show that on this assumption the maximum loss of height for an aircraft of speed V flying into an airstream of speed v flowing perpendicularly over a ridge of

/height

height h (above the surrounding countryside) would be $vh/(V-v)$. It could then be argued that the usual 1,000 feet adopted as the acceptable clearance over level country should be increased by values for the possible loss of height calculated from this relation. For an aircraft crossing a ridge obliquely, e.g. on a course making an angle θ with the ridge, the maximum involuntary loss of height would be $vh/(V\sin\theta - v)$. The following table gives the result of applying the criterion to a few typical cases of flight perpendicularly across a ridge, assuming an aircraft speed of 150 knots.

Height of Ridge (feet)	Wind Speed at Crest Level (knots)	Required increase in clearance (feet)	Clearance (feet)
1000	30	250	1250
2000	40	730	1730
3000	50	1500	2500
4000	60	2670	3670

The increases in safety height required for slower aircraft or for oblique flight across the ridge would, of course, be greater. It is clear, even from this elementary approach, that it may sometimes be impossible for a slow aircraft with small climbing power to cross a mountain range especially obliquely in strong winds. It is also clear that all of the factors, wind speed, aircraft speed, height of mountain and the direction of flight are significant. It follows that the tentative suggestions made by the Court of Enquiry which investigated the Aer Lingus aircraft accident in January 1952 cannot be regarded as a final solution since the proposed increases of clearance depend on only one of the important factors, viz. wind speed. An acceptable solution must also take account of the height of the mountain range above the surrounding terrain, must provide for the fact that slow aircraft would remain in regions of sink for longer than fast aircraft, and must in addition take account of the course of the aircraft.

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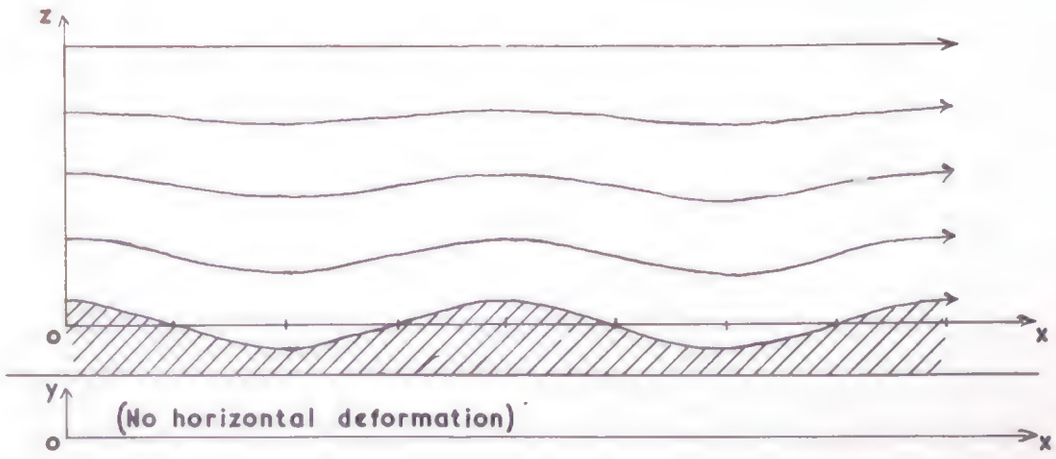


Fig. 1(a). $k > l_s$. Small scale corrugation (wavelength $< 2\pi$ km.)

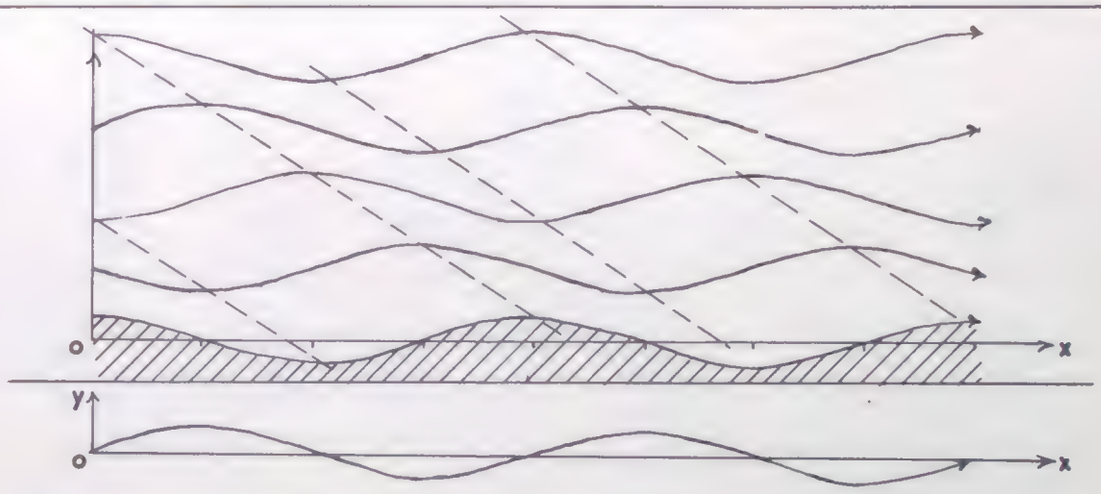


Fig. 1(b). $l_s > k > l_f$. Medium scale corrugation (wavelength 2π to 200π km.)

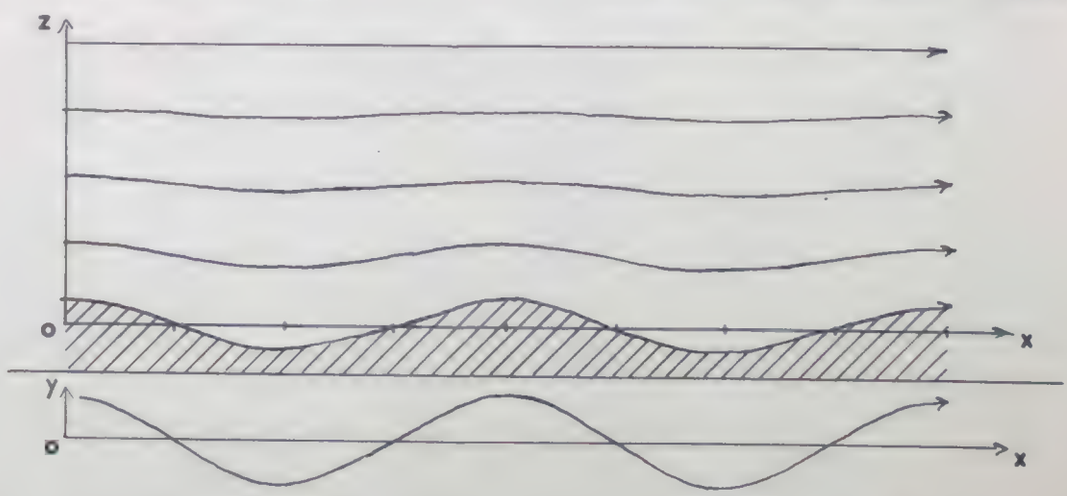


Fig. 1(c). $k < l_f$. Large scale corrugation (wavelength $> 200\pi$ km.)

Fig. 1. Streamlines produced by an infinite sinusoidal ground profile.

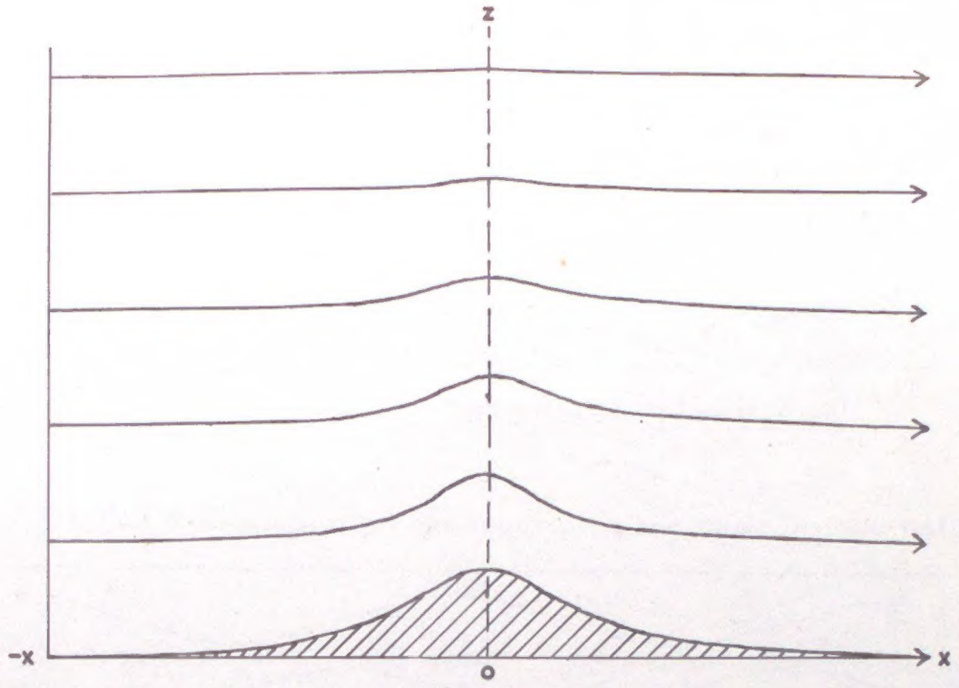


Fig. 2. The flow over a very small ridge of width up to about 100 m.

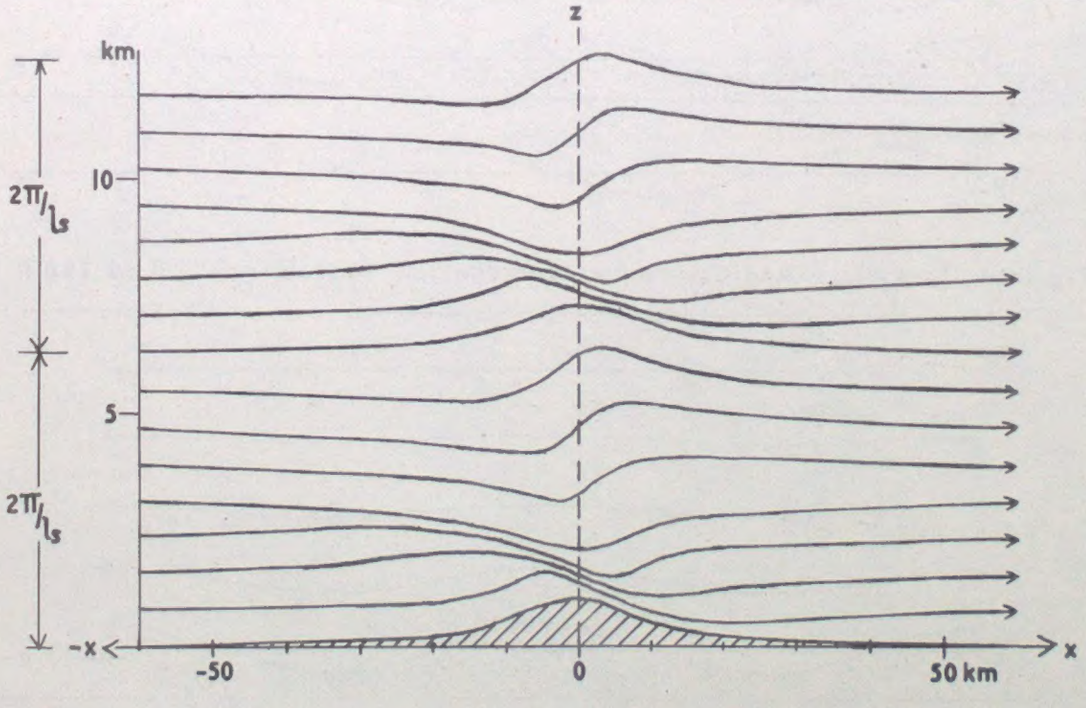
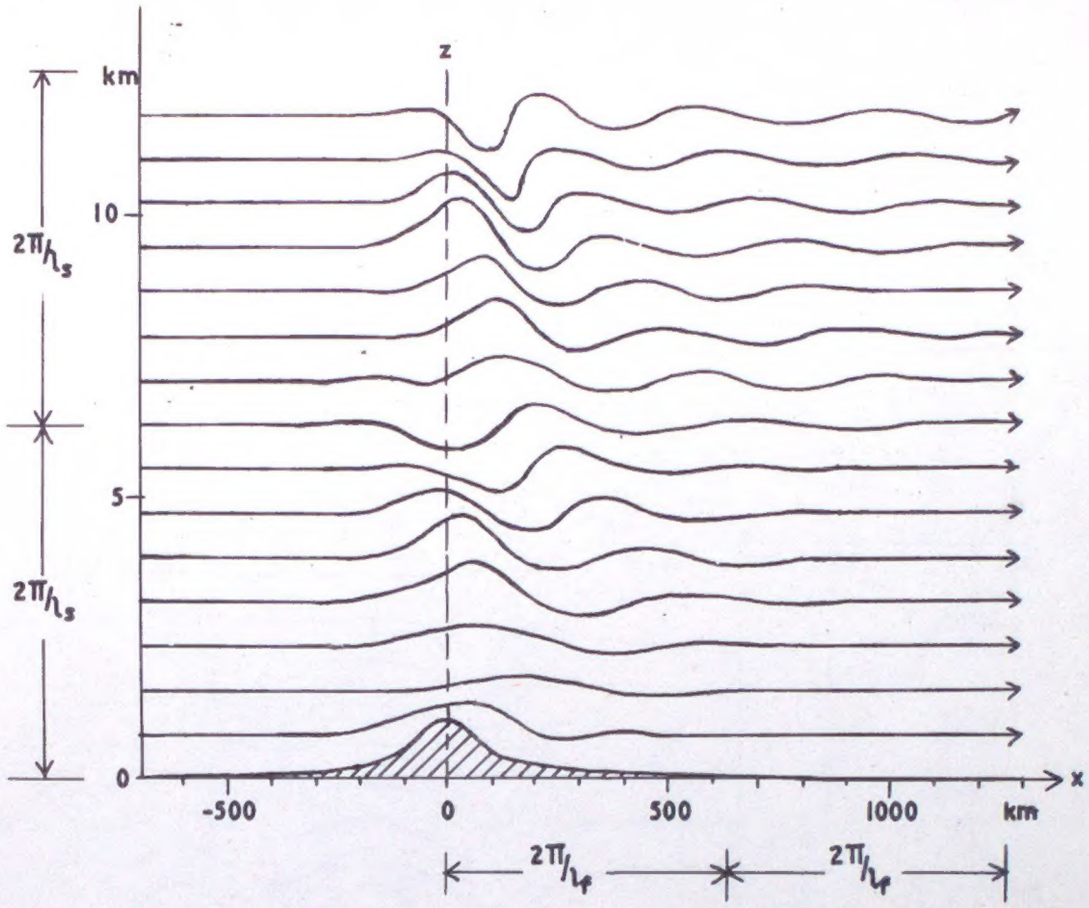
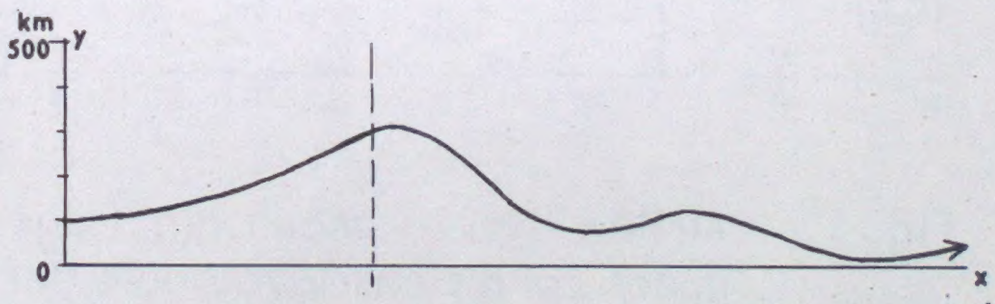


Fig. 3. The flow of a uniform airstream over a ridge having a width up to a few km. (after Queney). In the case illustrated $l_s = 1\ km^{-1}$, $a = 10$ km, $U = 10$ m/s.



(a) Vertical Projection of Streamlines.



(b) Horizontal Projection of Ground Streamline.

Fig. 4. The flow of a uniform airstream over a ridge having a width of about 100 km. In the case illustrated $l_s = 1\,km^{-1}$, $a = 100$ km., $l_f = \cdot 01\,km^{-1}$, $U = 10$m/s.

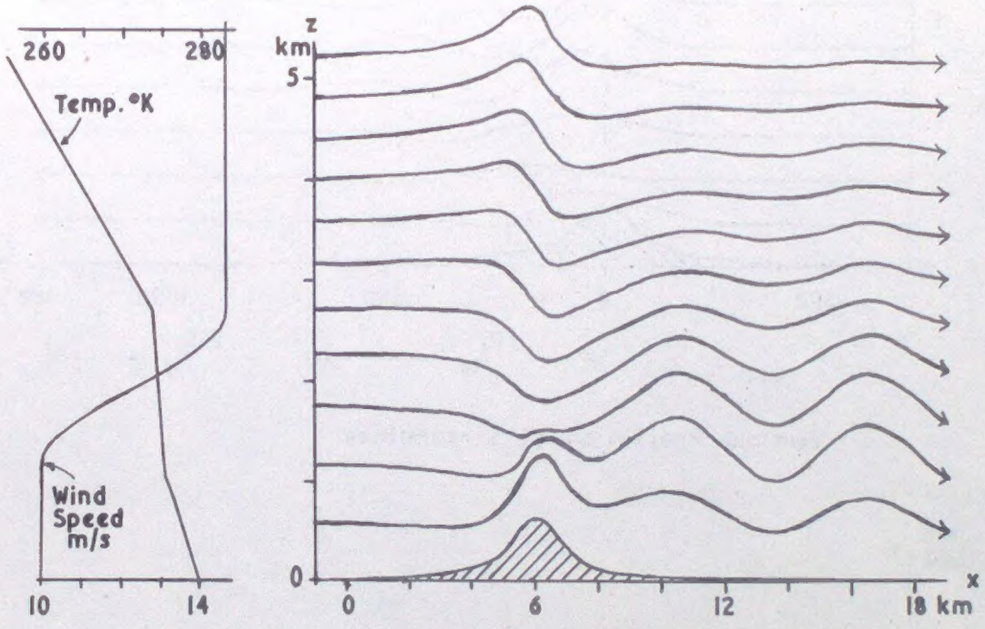
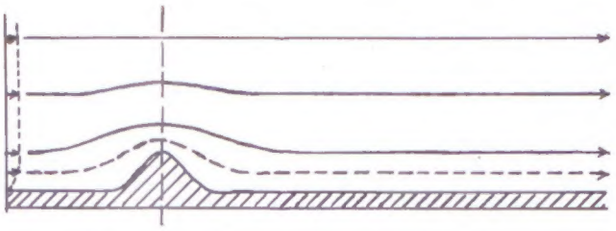
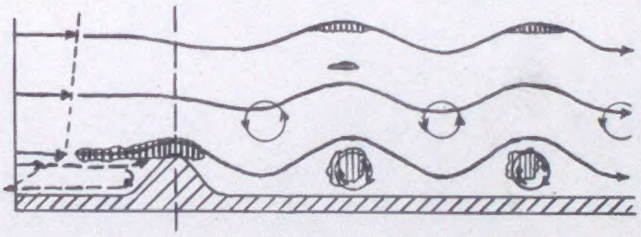


Fig. 5. Airflow over a mountain ridge under conditions favourable for the occurence of lee waves (after Scorer).

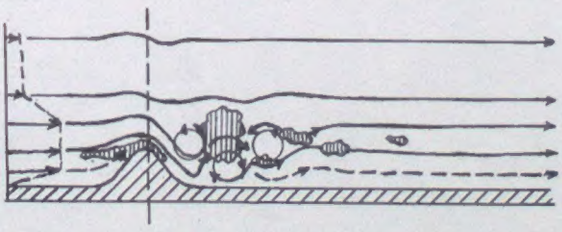


(a) Laminar streaming.

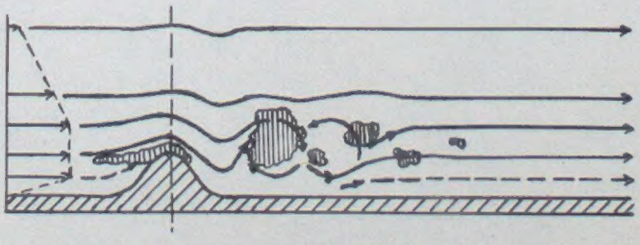
(b) Standing eddy streaming.



(c) Wave streaming.



(d) Rotor streaming.



(e) Rotor streaming

Fig. 6. Types of airflow over ridges. (after Förchtgott).